Enumeration of chinampas

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What is a chinampa?

The following graphs originated from problems in neuroscience. Consider the vertices with integer coordinates on the first quadrant of the plane.

- Select N vertices and call them *primary vertices*, see the vertex with double circle of Figure 1.
- Starting from left to right, bottom to top: if the vertex (x,y) and the vertex (x+1,y) are selected, then select the vertex (x+1,y+1) and add edges from (x,y), (x+1,y) to (x+1,y+1). Call these new vertices *secondary vertices*, see the vertex with a simple circle of Figure 1.

A chinampa is any connected component of the graph created with the previous instructions.

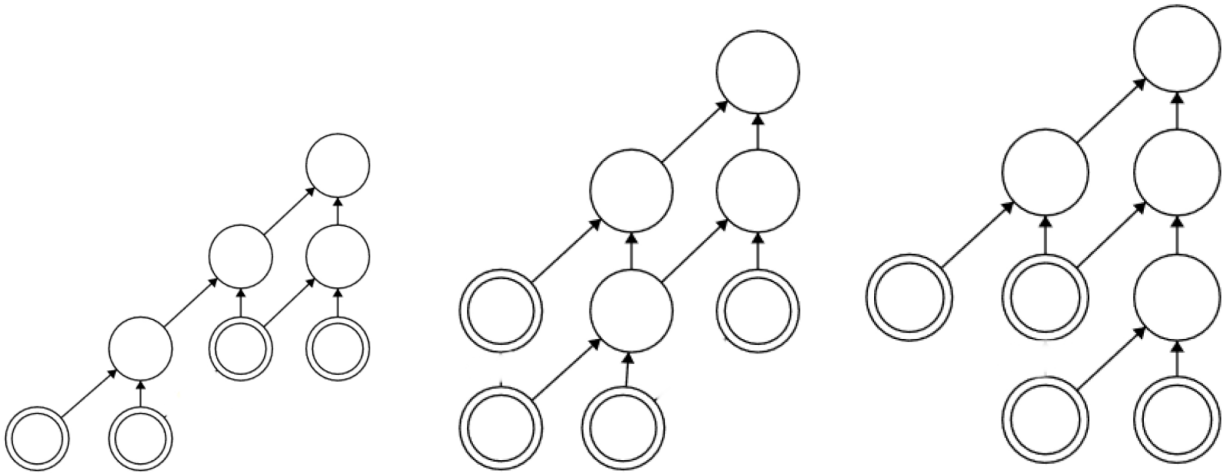

Figure 1: Three chinampas, in which primary vertices have double circles and secondary vertices have simple circles.

The *profit* is the number of secondary vertices minus the number of primary vertices.. For example, all chinampas of Figure 1 have profit 0. The *size* is the maximum of the height and the width of the chinampa. All chinampas in Figure 1 have size 4.

What does a standard chinampa look like?

When the size of the chinampa grows, we believe they resemble Figure 2.

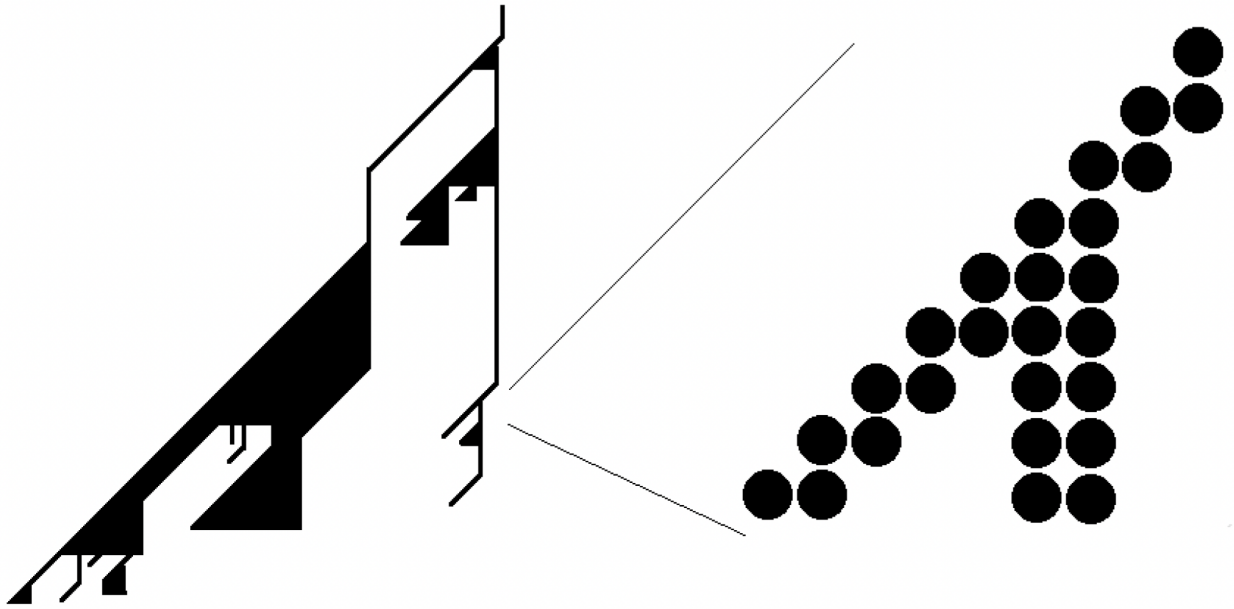

Figure 2: On the left a chinampa. On the right, we zoom in to see the nodes of the chinampa. For simplicity, we don't display the edges or differentiate between primary and secondary vertices.

Open problem:

Count the possible chinampas for any size and any profit. The cases of profit 0 and profit 1 have been solved for all sizes using generating functions. Enumerating certain families of chinampas is equivalent to finding order polynomials of posets. The main issue with profit >1 is that the roots on Figure 2 may touch, and then the profit jumps.

Why chinampas?

"The name is due to the similarity of the figures with an ancestral Mexican agricultural technique that uses soil to grow crops on a lake. We imagine that chinampas have crops above the soil, and underneath, there are roots."[1] The roots are pushed by the water and so they all move in the same direction.

Figure 1 and Figure 2 are taken from [1].

For more information:

[1] Dolores-Cuenca, Eric, José Antonio Arciniega-Nevárez, Anh Nguyen, Amanda Yitong Zou, Luke Van Popering, Nathan Crock, Gordon Erlebacher, and Jose L. Mendoza-Cortes. 2023. "Polychrony as Chinampas" *Algorithms* 16, no. 4: 193. https://doi.org/10.3390/a16040193